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ESTIMATION OF STRENGTH-REDUCTION FACTORS FOR ELASTOPLASTIC SYSTEMS: A NEW APPROACH

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SUMMARY

A new rule is presented to estimate strength-reduction factors in single-degree-of-freedom elastoplastic oscillators, in which reduction factors depend only on displacement elastic spectra, so they are not an explicit function of structural period. It is found that the proposed rule yields good results when applied to accelerograms recorded in firm and soft soils, and for damping ratios of 2, 5 and 10 per cent of the critical. It is shown that the proposed reduction rule is more general than others previously published, and at least as accurate. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: strength-reduction factors; elastoplastic systems

INTRODUCTION

Contemporary seismic design of buildings requires, among many other things, the estimation of various structural response parameters in the inelastic range. For instance, since enough strength must be furnished to limit ductility demand, μ, to a specified value, and thus prevent collapse, estimation of μ for a given strength is essential in the design process.

Estimation of required strengths, ductility demands or inelastic displacements is usually done using *strength-reduction factors*, R_μ. For an elastoplastic single-degree-of freedom oscillator subjected to a given ground motion, R_μ is the ratio between the strength required for elastic behaviour and the strength for which ductility demand equals μ. If $F(T,\mu)$ is the spectrum of required strengths, then

$$R_\mu(T) = \frac{F(T,1)}{F(T,\mu)} \tag{1}$$

where T is the structural period. Thus if $R_\mu(T)$ is known, then the required strength to attain a ductility demand μ can be obtained dividing the corresponding elastic force spectrum by $R_\mu(T)$. Also, it can be shown that the inelastic displacement for given ductility and period, $D(T,\mu)$, can be computed with

$$D(T,\mu) = D(T)\frac{\mu}{R_\mu(T)} \tag{2}$$

where $D(T)$ is the elastic relative displacement spectrum. Therefore, determination of $R_\mu(T)$ allows also computation of inelastic displacements from their elastic counterparts.

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There are limits imposed by structural dynamics to the strength-reduction factors: if $T \to 0$, $R_\mu(T) \to 1$ for all ductilities and dampings; also, if $T \to \infty$, $R_\mu(T) \to \mu$ for all ductilities and dampings. For other periods, however, there are no theoretical indications as to the values of $R_\mu(T)$, and many efforts, all empirical in nature, have been made to give rules to obtain strength-reduction factors. An excellent and thorough review of these efforts can be found in the work by Miranda and Bertero.[1]

Perhaps the most widely accepted reduction rule is the one by Newmark and Hall[2] which, in simplified forms, is presently included in many building codes to obtain $R_\mu(T)$. This rule specifies that, for periods not excessively short, $R_\mu = \mu$, which implies that elastic and inelastic displacements are equal, independently of ductility demand, as shown by equation (2).

Before 1985, all published reduction rules to estimate $R_\mu(T)$ were developed using relatively small collections of ground motions recorded in firm ground. After analysing the accelerograms obtained during the 19 September 1985 ($M_s = 8{\cdot}1$) Michoacán earthquake at several very soft sites in Mexico City, Meli and Ávila[3] observed, for the first time as to the authors' knowledge, that for certain periods the R_μ values were substantially higher than those predicted by Newmark and Hall's rule. This can be appreciated in Figure 1, which shows $R_\mu(T)$ for $\mu = 4$ for the SCT recording (EW component) of the 1985 Michoacán earthquake, along with an estimation using Newmark and Hall's[2] rule. As noted also by other authors (e.g. Reference 4) it is clear that $R_\mu(T)$ is underestimated by Newmark and Hall's rule for periods around 2 sec. Rosas *et al.*[5] analysed many accelerograms recorded at Mexico City's soft soils and concluded that, indeed, $R_\mu(T)$ values at certain periods were systematically higher than those predicted by Newmark and Hall's rule.

Miranda[6] examined numerous seismic recordings and proposed empirical site-dependent reduction rules. He showed that for soft soils, strength reductions depend systematically on the ratio T/T_s, where T_s is the period for which the 5 per cent damped elastic input-energy spectrum is maximum. It is worth noting that the reduction rule proposed by Miranda for soft sites is very different from those applicable for hard sites.

In this paper, a new rule to estimate $R_\mu(T)$ is presented, that can be applied to a wide variety of site conditions. With this rule, $R_\mu(T)$ is not an explicit function of structural period—as in all the rules previously published in the literature—but a function of elastic spectral displacement. This implies that the variation of $R_\mu(T)$ with T is mainly controlled by the shape of the displacement spectrum and not by a standard shape obtained empirically. The effect of site conditions, recognized as a crucial factor in determining $R_\mu(T)$, is therefore automatically included in the rule by means of the shapes of the elastic displacement spectra.

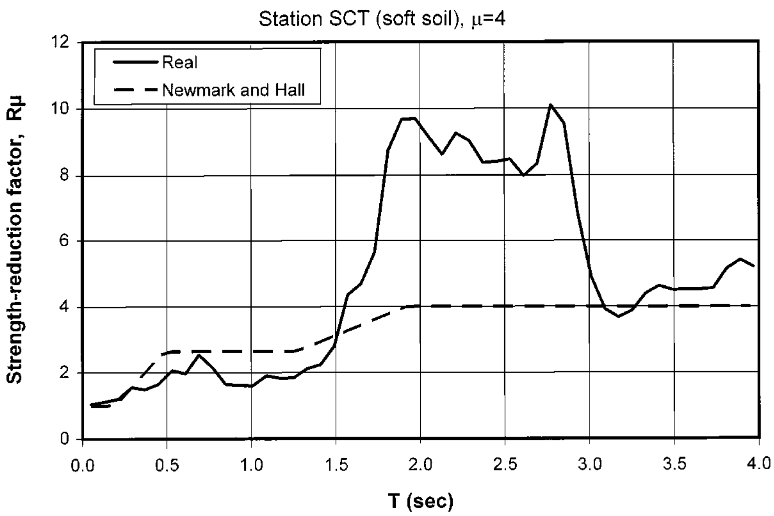

Figure 1. Solid line: R_μ spectrum for $\mu = 4$, computed for the 19 September 1985 SCT recording (EW component) in Mexico City. Dashed line: estimated R_μ with Newmark and Hall's rule

PROPOSED REDUCTION RULE

Figure 2 shows $R_\mu(T)$ for $\mu = 2$ for two accelerograms of very different nature, both recorded during the April 25, 1989, $M_s = 6{\cdot}9$ Guerrero earthquake. The first (Figure 2(A)) was obtained at station 01 (epicentral distance = 300 km, Mexico City's lake-bed zone) and the second (Figure 2(B)) was recorded at station PARS (epicentral distance = 70 km, firm ground). R_μ values are compared with the corresponding relative displacement and velocity elastic spectra, normalized with respect to peak ground values ($D(T)/D_{\max}$ and $V(T)/V_{\max}$, respectively). A coincidence can be appreciated in Figure 2 regarding the shapes of functions $D(T)/D_{\max}$, $V(T)/V_{\max}$ and R_μ: when the spectral levels are low (at short period) R_μ is also low; at long period, when $D(T)/D_{\max}$ and $V(T)/V_{\max}$ tend to 1 (since $D(T) \to D_{\max}$ and $V(T) \to V_{\max}$)R_μ tends to μ; and the reductions peak when $D(T)$ or $V(T)$ are close to their maxima. For comparison, predicted R_μ values using Newmark and Hall's rule are also shown.

It can be noticed in Figure 2 that, in fact, $R_\mu(T)$ varies considerably with T, and the differences between reductions at firm ground and at soft ground are evident. Newmark and Hall's rule works much better for PARS station (firm ground) than for station 01 (soft soil). But for both sites, most of the characteristics of the relative velocities and displacements are reflected in the R_μ spectra. Based on this similarity, the following reduction rule is proposed:

$$R_\mu(T) = 1 + \left(\frac{V(T)}{V_{\max}}\right)^{\alpha} \left(\frac{D(T)}{D_{\max}}\right)^{\beta} (\mu - 1) \tag{3}$$

It can be verified that this expression has correct limits for short and long period, regardless of the values of α and β. We do not have theoretical indications as to the values of these parameters, nor we know whether $R_\mu(T)$ could be better predicted by $D(T)/D_{\max}$ or by $V(T)/V_{\max}$, although, for a wide range of periods, these two quantities are strongly correlated, so one of them should be irrelevant. Therefore, the answers must be found by empirical means.

STATISTICAL DATA ANALYSIS

Four hundred and forty-five free-field accelerograms were analysed, recorded by the Guerrero Accelerographic Array (operated by the Institute of Engineering-UNAM and the University of Nevada-Reno) and the

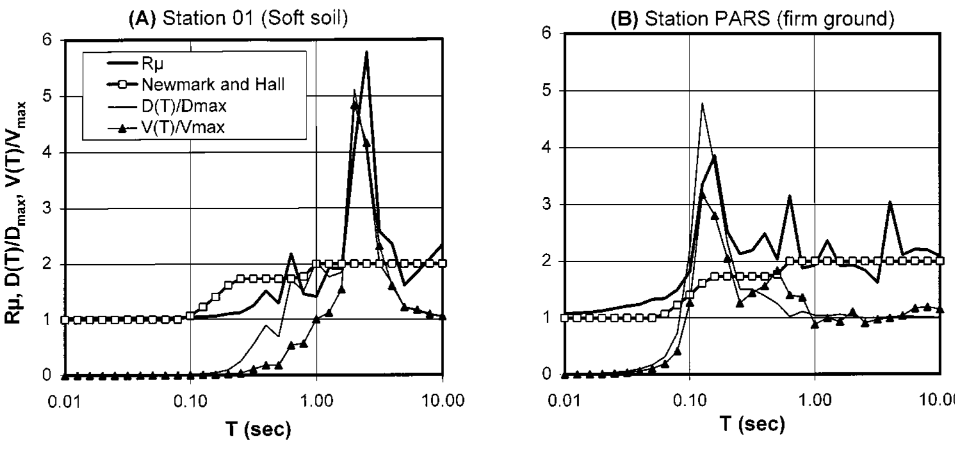

Figure 2. R_μ spectra computed for two accelerograms at soft soil (A) and firm ground (B). They are compared with their corresponding elastic displacement and velocity spectra, normalized with respect to peak ground values, $D_{\max}$ and $V_{\max}$, respectively. Estimations of R_μ with Newmark and Hall's rule are also presented for comparison

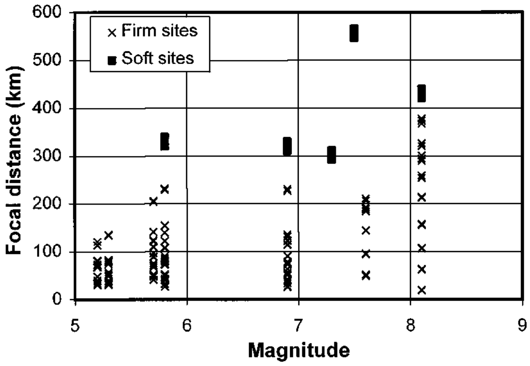

Figure 3. Magnitude–focal distance distribution of the strong motion recordings used in this study. All soft-soil accelerograms are from Mexico City

Mexico City Accelerographic Array (operated by the Institute of Engineering-UNAM, CENAPRED and Centro de Instrumentación y Registro Sísmico AC). Figure 3 shows a magnitude–focal distance plot of the strong-motion recordings used in this study. Magnitudes (M_s) go from 5·2 to 8·1, while focal distances vary from 16 to 550 km. These recordings cover a great variety of site conditions (from bedrock to lake-bed sites).

All ground motions used in this study were digitally recorded with instruments of 12, 18 or 19 bits, so the quality of the recordings is very good. No base-line corrections had to be made, except to compensate for a constant offset in a few accelerograms. Recordings were high-pass filtered (Butterworth, 4 poles) to eliminate static components in ground velocity. The corner frequency of the filter varied depending on size and duration of each time history, but the largest corner frequency used was 0·08 Hz (12 sec). If high-pass filtering at this frequency was not enough to remove the static component in the velocity time history, the accelerogram was disregarded. Elastic (absolute acceleration, relative velocity and relative displacement) and inelastic ($\mu = 1{\cdot}5$, 2, 4 and 8) response spectra were computed for each accelerogram, for $\zeta = 5$ per cent of the critical damping, for 31 periods in the period range 0·01 sec $< T <$ 10 sec. From this information, R_μ spectra were computed.

Accelerograms were processed in three different groups. The first (152 time histories) includes only recordings at firm sites; the second (293 time histories) includes only recordings obtained at soft sites; and the third group comprises all 445 accelerograms. For each group of accelerograms and for each ductility, values of α and β were found that minimized the root-mean-square logarithmic error, σ, defined as

$$\sigma^2 = \frac{1}{N}\frac{1}{M}\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\log\frac{R_{ij}}{R^*_{ij}}\right)^2 \tag{4}$$

where N is the number of recordings analysed, M is the number of structural periods for which R_μ values were computed, and R_{ij} and R^*_{ij} are, respectively, the real and computed values of R_μ for recording i and structural period j. R^*_{ij} values were computed with equation (3) for various values of α and β and, for each case, those yielding the minimum value of σ were selected. It was chosen to minimize the logarithmic error since R_μ factors are generally used multiplying or dividing other quantities, so an indication of the uncertainty in the ratio between real and predicted values is usually of more interest. For small values of σ, this quantity is proportional to the coefficient of variation (actually, if we had used natural instead of common logarithms, σ would be almost equal to the coefficient of variation). To gain sensitivity as to the values of σ, one can recall that with high probability (68 per cent, if the logarithmic error had normal distribution), the true value of R_μ would fall between $10^{-\sigma}$ and 10^{σ} times the estimated value.

Values of α and β were obtained for three regression models: model A, where $R_\mu(T)$ is assumed to depend only on $V(T)/V_{\max}$, which amounts to fix $\beta = 0$; model B, where $R_\mu(T)$ is assumed to depend only on

$D(T)/D_{max}$, that is, fixing $\alpha = 0$; and model C, where $R_\mu(T)$ is assumed to depend on both $V(T)/V_{max}$ and $D(T)/D_{max}$. Results of the statistical analysis are presented in Tables I–III.

SELECTION OF REGRESSION MODEL

Several observations can be made from Tables I–III

(1) For the three groups of accelerograms and all values of ductility, the smallest σ is obtained with model C, this is, when $R_\mu(T)$ is assumed to depend on both $V(T)/V_{max}$ and $D(T)/D_{max}$. However, the best model is not necessarily the one with the smallest standard error, but one that combines simplicity (a small number of parameters) and accuracy. This is the case, for instance, of results for $\mu = 1{\cdot}5$, where for the three groups of accelerograms, σ values obtained for model A ($R_\mu(T)$ depends only on $V(T)/V_{max}$) are only marginally larger than those obtained for model C.

Table I. Results of model A ($R_\mu(T)$ depends only on elastic spectral velocities)

Accelerogram group	Parameter	Ductility 1·5	2	4	8
Firm sites	α	0·436	0·507	0·609	0·680
	β	0·000	0·000	0·000	0·000
	σ	0·043	0·062	0·104	0·151
Soft sites	α	0·403	0·443	0·531	0·609
	β	0·000	0·000	0·000	0·000
	σ	0·041	0·059	0·110	0·158
All sites	α	0·407	0·451	0·542	0·621
	β	0·000	0·000	0·000	0·000
	σ	0·042	0·060	0·109	0·155

Table II. Results of model B ($R_\mu(T)$ depends only on elastic spectral displacements). Numbers in boldface correspond to the preferred model (see text ahead)

Accelerogram group	Parameter	Ductility 1·5	2	4	8
Firm sites	α	0·000	0·000	0·000	0·000
	β	0·303	0·357	0·440	0·500
	σ	0·052	0·072	0·091	0·096
Soft sites	α	0·000	0·000	0·000	0·000
	β	0·359	0·397	0·479	0·551
	σ	0·044	0·059	0·092	0·113
All sites	α	0·000	0·000	0·000	0·000
	β	**0·348**	**0·388**	**0·472**	**0·543**
	σ	0·048	0·065	0·094	0·110

Table III. Results of model C ($R_\mu(T)$ depends on both elastic spectral displacements and velocities)

Accelerogram group	Parameter	Ductility 1·5	2	4	8
Firm sites	α	0·440	0·408	0·280	0·163
	β	− 0·002	0·073	0·240	0·387
	σ	0·043	0·059	0·081	0·093
Soft sites	α	0·257	0·094	− 0·171	− 0·359
	β	0·131	0·313	0·631	0·869
	σ	0·041	0·058	0·092	0·113
All sites	α	0·328	0·242	0·105	0·009
	β	0·069	0·182	0·382	0·536
	σ	0·042	0·059	0·092	0·110

(2) From Table III it can be noticed that, for all ductilities, α and β differ considerably depending on the group of accelerograms used. This means that the relative influence of $D(T)$ and $V(T)$ in predicting $R_\mu(T)$ is different for firm sites than for soft sites. Then, although on average model C would be better to predict — in the sense of a smaller standard error—it does not seem robust enough.
(3) For models A and B, α and β values do not differ much depending on accelerogram group, that is, depending on site conditions. This gives generality and robustness to models A and B.
(4) For the small ductilities, $R_\mu(T)$ is better predicted by model A (velocity-dependent) whereas for the larger ductilities a best fit is obtained with model B (displacement-dependent). However, a combined model (A for small ductilities, B for larger) would be discontinuous, since a boundary between 'small' and 'large' ductility would have to be drawn.

These observations led us to disregard model C. In view of the fact that σ values associated to model B (displacement-dependent) are more uniform (they change less with ductility), this model was selected as the preferred one for all ductilities and accelerogram groups, with the β values given in Table II for the accelerogram group that includes all recordings processed (boldface numbers in Table II).

It can be noticed that the model, so far, is discontinuous, in the sense that β values have been determined only for four selected ductilities. In this form, the model cannot be used, for instance, to solve the inverse problem, that is, to find the ductility demand given the value of R_μ. To solve this problem, a continuous variation of β with μ has to be constructed. Figure 4 shows computed β values as a function of μ (triangles), along with a continuous approximation, which has the following analytical form:

$$\beta(\mu) = 0{\cdot}388(\mu - 1)^{0{\cdot}173} \tag{5}$$

Small errors are induced using this approximation instead of the computed point values.

In summary, after analysing the statistical results, and the advantages of the various regression models examined, the proposed model to estimate $R_\mu(T)$ is given by

$$R_\mu(T) = 1 + \left(\frac{D(T)}{D_{\max}}\right)^{\beta(\mu)} (\mu - 1) \tag{6}$$

with $\beta(\mu)$ given in equation (5).

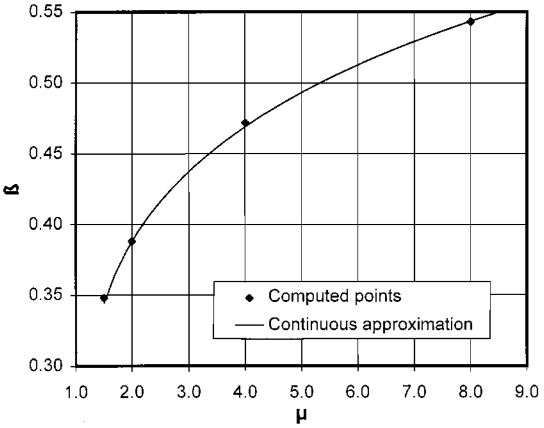

Figure 4. Dots: β values computed for the ductilities, μ, examined (1·5, 2, 4 and 8). Solid line: continuous approximation of β as a function of μ

PERFORMANCE OF THE PROPOSED RULE

Figure 5 shows real and predicted R_μ spectra for ductilities 2 (top row) and 8 (bottom row) for six selected firm-ground recordings. Predicted $R_\mu(T)$ have been computed with the model described by equations (5) and (6); note that the fit is satisfactory. Also, predicted $R_\mu(T)$ closely follow details of the real ones, which would have not occurred had standard reduction rules been used.

Figure 6 shows real and predicted R_μ spectra for six soft soil recordings. The fit is, again, satisfactory, even when the shapes of R_μ spectra in soft soil differ considerably from those at firm sites and among themselves (compare, for instance, results for recordings 5425 NS and 8025 EW in Figure 6). It is worth recalling that predicted R_μ spectra for soft soils have been computed using exactly the same model used for firm sites (equations (5) and (6)).

It would be very difficult to present direct comparisons between real and predicted R_μ values for many accelerograms. Instead, the accuracy of the proposed rule will be compared with that of some selected reduction rules previously published in the literature. There is not, to the authors' knowledge, another reduction rule general enough to be used for both firm and soft soils. In view of this, comparisons were made with different rules depending on soil type.

For firm sites, two recently developed reduction rules were used. The first, by Nassar and Krawinkler,[8] for 5 per cent damping, is given by

$$R_\mu(T) = (1 + C(T)(\mu - 1))^{1/C(T)} \tag{7}$$

$$C(T) = \frac{T}{1+T} + \frac{0{\cdot}42}{T} \tag{8}$$

The second rule, by Miranda,[6] is the following:

$$R_\mu(T) = 1 + \frac{(\mu - 1)}{\Phi(T)} \tag{9}$$

$$\Phi(T) = 1 + \frac{1}{12T - \mu T} - \frac{2}{5T}\exp(-2(\ln T - 0{\cdot}2)^2) \tag{10}$$

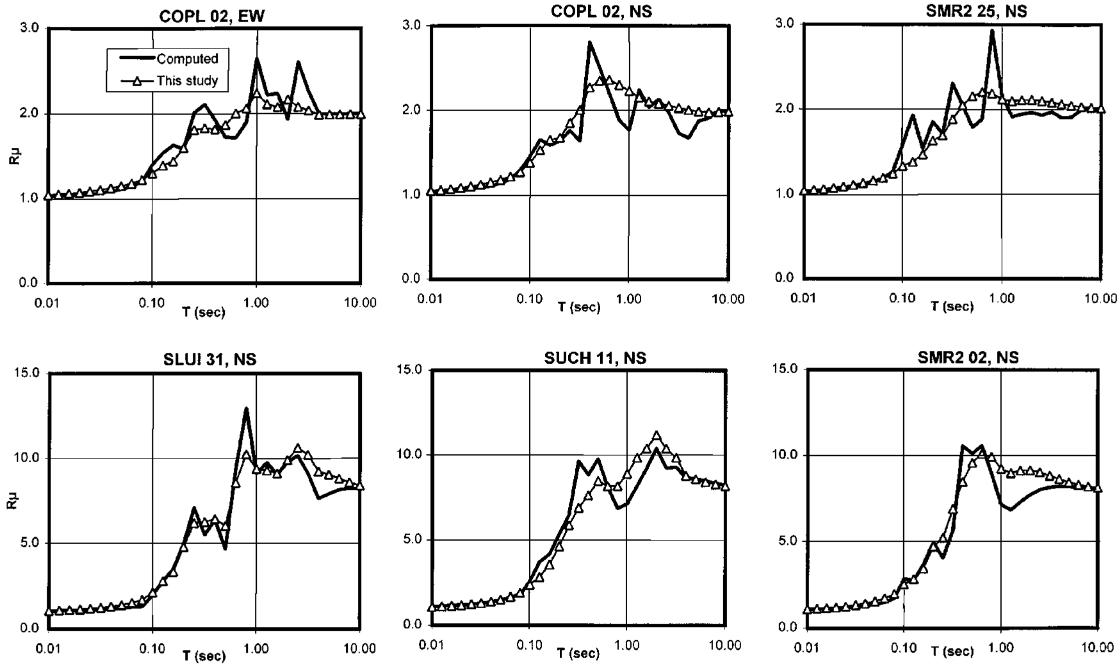

Figure 5. Continuous line: real R_μ spectra for $\mu = 2$ (top row) and $\mu = 8$ (bottom row), for several firm gound recordings. Triangles: R_μ spectra computed with the rule proposed in this study

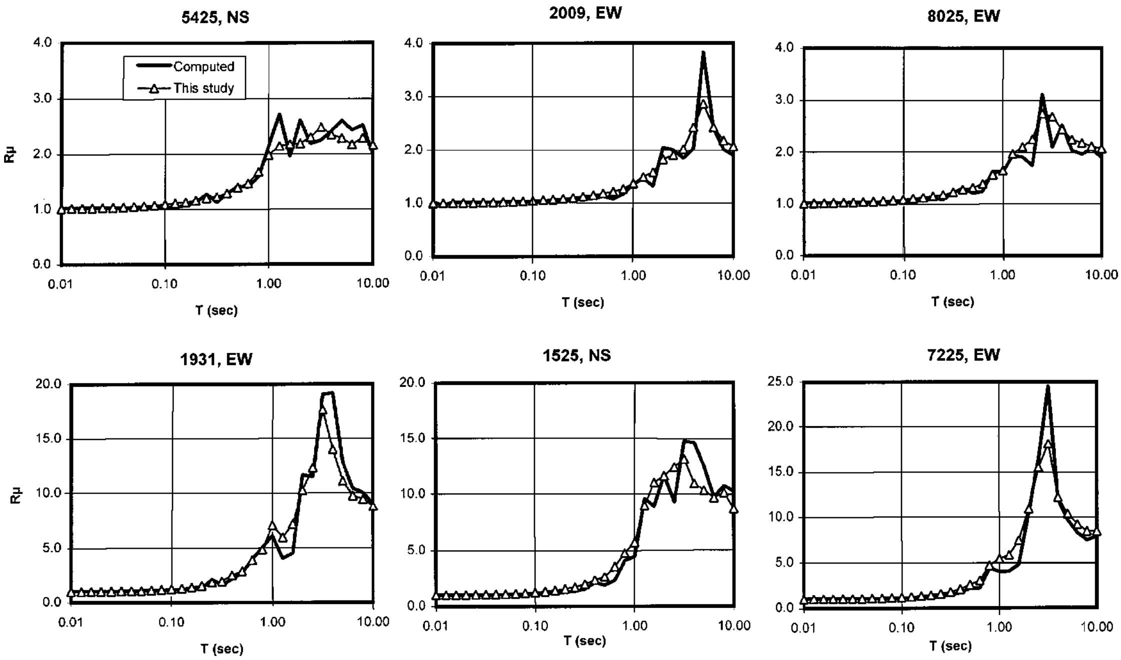

Figure 6. Continuous line: real R_μ spectra for $\mu = 2$ (top row) and $\mu = 8$ (bottom row), for several soft-soil recordings. Triangles: R_μ spectra computed with the rule proposed in this study

For soft soils, the rule by Miranda *et al.*[7] was used. Here, $R_\mu(T)$ is also given by equation (9), but with a different expression for $\Phi(T)$:

$$\Phi(T) = 1 + \frac{1}{1{\cdot}8(T/T_s)} - 3{\cdot}48\exp\left(-2{\cdot}17\left(\frac{T}{T_s} - 0{\cdot}95\right) - \exp\left(-2.17\left(\frac{T}{T_s} - 0.95\right)\right)\right) \tag{11}$$

where T_s, as mentioned earlier, is the motion's predominant period. In Miranda's[6] original definition, T_s is the period for which the input-energy spectrum is maximum. It was found that, although this definition works well for many recordings, it fails for sites with very long predominant period ($T_s > 3$ sec). In these cases, T_s was selected as the period that produced the smallest average estimation error for each recording.

$R_\mu(T)$ were computed for all accelerograms, at 31 periods in the range $0{\cdot}01 \leqslant T \leqslant 10$ sec, using the various reduction rules. For each computed R_μ value, the estimation error was obtained as the common logarithm of the ratio between real and computed values. The cumulative distributions of estimation errors (the probability of an error being lower than a specified value) are presented in Figure 7, for the four ductilities examined and, separately, for firm and soft soils. These results are summarized in Tables IV and V, where values of the error associated to percentiles 16, 50 and 84, E_{16}, E_{50} and E_{84}, respectively, are presented.

E_{50} can be considered a measure of the average estimation error and is also a measure of bias: the estimation is unbiased if $E_{50} = 0$. It can be appreciated in Tables IV and V that, for most of the cases, the proposed rule is almost unbiased. Exceptions are the cases of $\mu = 4$ and 8 in soft soils, where $E_{50} = -0{\cdot}04$ and $-0{\cdot}05$, respectively, which means that, on average, R_μ factors will be underestimated by factors of $10^{-0{\cdot}04} = 0{\cdot}91$ for $\mu = 4$ and $10^{-0{\cdot}05} = 0{\cdot}89$ for $\mu = 8$. Note that an underestimation of R_μ means an overestimation of the required strength, so this is an error on the conservative side. In general, the proposed rule is less biased that the others examined. E_{16} gives a reasonable bound for the underestimation error that could take place, while E_{84} measures the largest overestimation of R_μ that could be committed. With the proposed rule, the largest value of E_{84} is 0·11 and occurs for $\mu = 4$ at first sites. In this case, a reasonable value for the largest overestimation of R_μ would be a factor of $10^{0{\cdot}11} = 1{\cdot}29$.

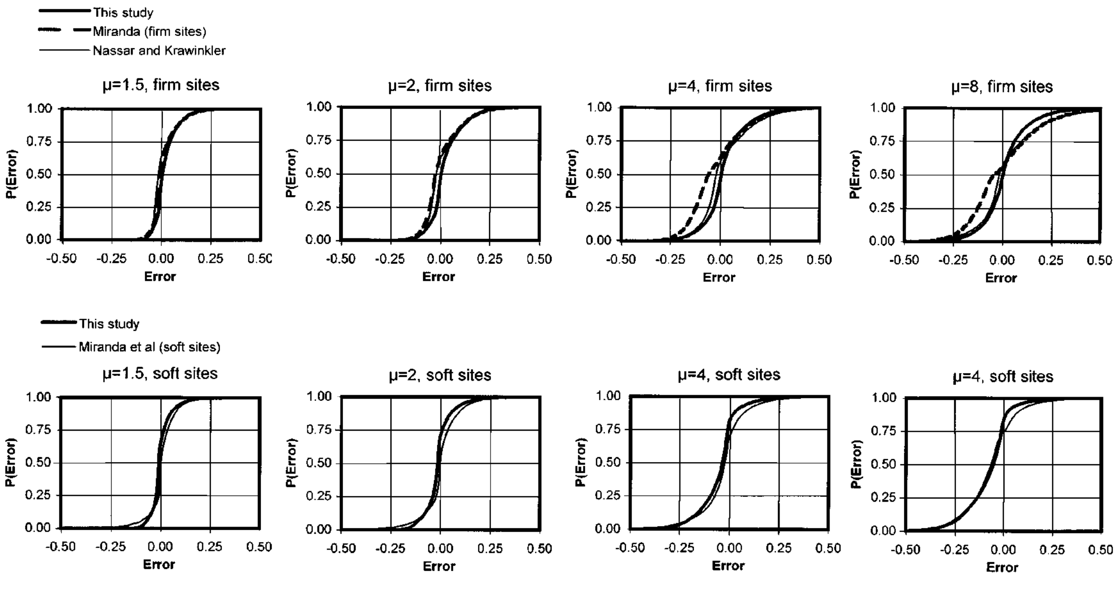

Figure 7. Probability distribution of the estimation error (probability that the error in the estimation of R_μ is lower than given values) for several ductilities, soil types and estimation rules

Table IV. Estimation errors for firm sites, associated to different percentiles, ductilities and reduction rules

μ	Rule	Percentile		
		16%	50%	84%
1·5	This study	− 0·02	0·01	0·07
	Miranda[6] (firm sites)	− 0·03	− 0·01	0·07
	Nassar and Krawinkler[8]	− 0·04	− 0·01	0·07
2	This study	− 0·04	0·00	0·10
	Miranda[6] (firm sites)	− 0·07	− 0·02	0·09
	Nassar and Krawinkler[8]	− 0·06	− 0·02	0·10
4	This study	− 0·06	0·00	0·11
	Miranda[6] (firm sites)	− 0·15	− 0·06	0·12
	Nassar and Krawinkler[8]	− 0·08	− 0·02	0·14
8	This study	− 0·08	0·00	0·10
	Miranda[6] (firm sites)	− 0·16	− 0·04	0·17
	Nassar and Krawinkler[8]	− 0·10	− 0·02	0·16

Table V. Estimation errors for soft sites associated to different percentiles, ductilities and reduction rules

μ	Rule	Percentile		
		16%	50%	84%
1·5	This study	− 0·03	− 0·01	0·04
	Miranda[6] (soft sites)	− 0·03	0·00	0·06
2	This study	− 0·06	− 0·01	0·03
	Miranda[6] (soft sites)	− 0·05	0·00	0·07
4	This study	− 0·14	− 0·04	0·01
	Miranda[6] (soft sites)	− 0·11	− 0·02	0·05
8	This study	− 0·19	− 0·05	0·00
	Miranda[6] (firm sites)	− 0·18	− 0·06	0·04

Note in Tables IV and V that the estimation errors associated to the proposed reduction rule are, in many cases, the smallest. It must be said, however, that comparison of errors produced by the various reduction rules is not strictly fair, since the parameters of the proposed model have been determined to minimize these errors, while parameters of the other rules were obtained with different sets of strong-motion recordings. It is conceivable that if parameters of reduction rules by other authors were re-determined with the 445 accelerograms used in this study, their corresponding errors would be smaller, but, very likely, not much smaller. Therefore, it can be concluded that the proposed model is at least as accurate as the other rules examined. The proposed model, however, is more general, since it is applicable, with exactly the same parameters, for a wide variety of soil types.

Table VI. β values that minimize the estimation error, σ, for several damping ratios and ductilities; σ values are also included. Results correspond to all recordings processed

		Ductility			
Damping	Parameter	1·5	2	4	8
2%	β	0·355	0·395	0·481	0·556
5%	β	0·348	0·388	0·472	0·543
10%	β	0·337	0·377	0·457	0·519
Average	β	0·347	0·387	0·470	0·540
2%	σ	0·063	0·080	0·103	0·116
5%	σ	0·048	0·065	0·094	0·110
10%	σ	0·040	0·057	0·085	0·100

RESULTS FOR OTHER DAMPING LEVELS

Results presented so far correspond to a single-degree-of-freedom oscillator with 5 per cent of the critical damping. It is interesting to investigate the validity of the proposed model for other damping ratios. Two and 10 per cent values were chosen, since they reasonably cover the interest of structural engineering. Results show that the proposed model predicts with similar accuracy the R_μ spectra, as long as $D(T)$ in equation (6) are the spectra with the corresponding damping ratio, that is, 2 and 10 per cent in our case. For these two damping ratios, the β values that minimize the error are approximately the same than those obtained for the case of 5 per cent. These values, and the corresponding estimation errors are presented in Table VI; values associated to 5 per cent damping are also included for comparison.

In view of this, the proposed model (equations (5) and (6)) with damping-independent parameters can be safely used for damping ratios between 2 and 10 per cent, as long as $D(T)$ in equation (6) is computed for the appropriate level of damping.

DISCUSSION

Examination of the literature on the subject shows that the behaviour of R_μ factors has been considered so different depending on soil type that derivation of site-dependent rules has been justified. It is not clear, however, where the boundary between 'soft' and 'firm' sites has to be drawn. Ordaz and Pérez-Rocha[9] attributed the difference of behaviour between firm and soft sites to the motion's bandwidth. This effect is still present in the proposed model, but it is now implicit in the shapes of the elastic displacement spectra. Also implicit is the dependence of $R_\mu(T)$ on the ratio T/T_s, but the rule proposed in this paper has the advantage that T_s is not used, so ambiguity in its definition is resolved. In consequence, the proposed model is useful for various site conditions, which, in a way, unifies the treatment of reduction rules.

Dependence of $R_\mu(T)$ on period, a fact documented long ago, remains in the proposed model, but in an implicit manner: $R_\mu(T)$ depends on period inasmuch as $D(T)$ depends on T. Furthermore, limits imposed by structural dynamics to $R_\mu(T)$ at very short and very long periods are correctly satisfied with the rule proposed.

As it can be noticed in equation (6), computation of $R_\mu(T)$ requires the knowledge of the peak ground displacement, D_{max}. Estimation of this ground motion parameter is also required in other rules. For instance, to apply Newmark and Hall's rule, D_{max} and V_{max} must be known (see, for instance, Reference 6. p. 361). The same happens with the rule by Riddell and Newmark.[10] D_{max} is sometimes difficult to estimate; its value may

change depending on the filters or the integration algorithm employed. However, when processing the 445 digital accelerograms used in this study, it was found that in most of the cases, computed D_{max} was reasonably stable. In the cases of unstable determinations of D_{max}, a tetralogarithmic representation of $D(T)$ was constructed and D_{max} was selected according to the observed spectral tendencies. In many cases, D_{max} can be taken as the spectral elastic displacement for long period ($T > 5$ sec at firm sites, $T/T_s > 4$ at soft sites).

R_μ factors predicted with the proposed rule are expected values. This means that errors can be made both for the safe side (underestimating R_μ) as well as for the unsafe side (overestimating R_μ). There are very few studies regarding the uncertainty associated to estimation of R_μ factors, which is rarely accounted for in determination of safety factors or risk evaluation. A full-fledged probabilistic analysis would treat R_μ as a random variable and would compute safety factors integrating with respect to all possible values of R_μ weighted by the corresponding probability density. A practical solution to this problem is to use R_μ factors that are associated to lower percentiles, that is, to higher probabilities of exceedance. For instance, the R_μ value associated to percentile 16 (84 per cent chance of exceedance) could be used. This value can be estimated by that predicted by the proposed rule multiplied by $10^{-\sigma}$, a factor that lies between 0·8 and 0·9, depending on ductility (see Table II).

As defined in this paper, R_μ factors are not design parameters but *response* parameters. They relate strength (a design parameter) with ductility demand (a response parameter; see equation (1)) or inelastic displacement (a response parameter) with ductility demand and strength (see equation (2)). Since many building codes admit that $R_\mu = \mu$ for a certain period range, it is easy to take μ for R_μ. Codes usually specify the maximum allowable ductility for a given structural system. In order to compute the appropriate strength to limit ductility demand to the specified value, R_μ factors should be computed with rules as accurate as possible. In many cases, as shown in this and other papers, the rule $R_\mu = \mu$ is not accurate enough.

Real buildings are not single-degree-of-freedom elastoplastic oscillators. Therefore, the discussion in the previous paragraph is a simplification of the actual problem. We believe, however, that it is important to study with detail the real behaviour of the model that, as simple as it is, is the core of our present design practices. This paper addressed the reduction of elastic response spectra to estimate inelastic response spectra. In order to reduce elastic design spectra to obtain inelastic design spectra, the results of this paper must be modified by taking into account at least the following aspects: structural overstrength, uncertainties on period estimation, uncertainties in estimation of R_μ itself, and multi-degree-of-freedom effects.

SUMMARY AND CONCLUSIONS

Based on the similarity between elastic displacement spectra, $D(T)$, and spectra of strength-reduction factors, $R_\mu(T)$, a rule to estimate this quantity has been proposed, in which $R_\mu(T)$ depends only on $D(T)$ and two empirically determined parameters (see equations (5)–(6) and Table II).

It has been shown that the proposed rule has appropriate limits for short and long period, and that is at least as accurate as other rules proposed in the literature. However, it is more general in two senses: (a) it applies for a wide variety of site conditions, which are implicit in the model since they are included in the shape of $D(T)$; and (b) it can be used for damping ratios between 2 and 10 per cent, as long as $D(T)$ is computed for the appropriate damping.

A physical interpretation of the proposed rule is unknown to the authors. Nevertheless, this is (temporarily, we hope) replaced by empirical confirmation of its adequacy when applied to more than 400 accelerograms of very different characteristics.

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